$T^i_j = \begin{cases} 0, & i \neq j \\ 2, & i = j \end{cases} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

№1.

$C = \begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix} \qquad C^{-1} = -\frac{3}{2}$

$(T^{i'}_{j'}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

№3.

$T^{2'2'}_{1'} \qquad T = e^1 \otimes e_2 \otimes e_2 + 2e^2 \otimes e_2 \otimes e_2 \qquad f_1 = 2e_1,\ f_2 = e_2$

$T^{22}_1 = 1$

ост-е $= 0$

$T^{22}_2 = 2$

$C^i_j = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$(C^{-1})^i_j = d^i_j = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$

$T^{2'2'}_{1'} = C^{2'}_i C^{2'}_j d^k_{1'} T^{ij}_k = C^2_2 C^2_2 d^1_{1'} \cdot T^{22}_1 = 1 \cdot 1 \cdot 1 \cdot 1 = 1$

№5.

T_{ijk} – кососим., $\mathbb{R}^3 \quad T_{123} = 1$

$f_1 = e_2$

$f_2 = -e_3$

$f_3 = e_1 + e_2$

$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = a^i_j$

$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix} = d^i_j$

$T_{1'2'3'} = d^i_{1'} d^j_{2'} d^k_{3'} T_{ijk} =$

$= d^1_{1'} d^2_{2'} d^3_{3'} T_{123} + d^1_{1'} d^3_{2'} d^2_{3'} T_{132} +$

$+ d^2_{1'} d^1_{2'} d^3_{3'} T_{213} + d^3_{1'} d^1_{2'} d^2_{3'} T_{312} +$

$+ d^2_{1'} d^3_{2'} d^1_{3'} T_{231} + d^3_{1'} d^2_{2'} d^1_{3'} T_{321} = 0 + 0 + 0 + (-1) \cdot T_{312} + 0 + 0 + = \underline{-1}$

$T_{312} = {}_\sigma T_{123} \qquad \sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad (-1)^\sigma = 1$

№7.

$Alt(\xi \otimes \eta)$, $\quad \xi = 2e_1 + e_2 \qquad \xi \otimes \eta = 4e_1 \otimes e_1 + 2e_2 \otimes e_1$

$\eta = 2e_1$

$Alt(\xi \otimes \eta) = \frac{1}{2!} \sum_\sigma (-1)^\sigma {}_\sigma(\xi \otimes \eta) = \frac{1}{2} \cdot 1 \cdot (4e_1 \otimes e_1 + 2e_2 \otimes e_1) -$

$- \frac{1}{2}(4e_1 \otimes e_1 + 2e_1 \otimes e_2) =$

$= e_2 \otimes e_1 - e_1 \otimes e_2$

№9

$Sym(\xi \otimes \eta) \qquad \xi = e_2 - e_1 \qquad \xi \otimes \eta = 4e_1 \otimes e_1 - 4e_2 \otimes e_1$

$\eta = -4e_1$

$Sym(\ ) = \frac{1}{2} \sum_\sigma \sigma(\ ) = \frac{1}{2}(4e_1 \otimes e_1 - 4e_2 \otimes e_1) + \frac{1}{2}(4e_1 \otimes e_1 - 4e_2 \otimes e_1) =$

$= 4e_1 \otimes e_1 - 2e_2 \otimes e_1 - 2e_1 \otimes e_2$

$\xi \otimes \eta (5e_1, 4e_2)$

№10.

$\xi = e^1 + e^2$

$\eta = e^1 - e^2$

$= 1$

№11

$v = 2e_1 - 3e_2$

$\xi = e^1 + e^2$

$C_1^1(v \otimes \xi) = 2 - 3 = -1$

№12.

$e_1 \otimes e_2 + e_1 \otimes e_3 \quad \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

$e_2 \otimes e_1 + e_3 \otimes e_1$

№13.

$e_3 \otimes e_2 \otimes e_1 + e_1 \otimes e_1 \otimes e_2 \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

$e_2 \otimes e_3 \otimes e_1 + e_1 \otimes e_1 \otimes e_2$

№14.

$T = 2e_1 \otimes e^2 + 4e_2 \otimes e^1 \qquad (g_{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad (g^{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}$

$T_2^1 = 2 \quad T_1^2 = 4 \qquad T^{ij} = g^{i\alpha} T_\alpha^j$

$T^{11} = g^{11} T_1^1 = \underline{0} \quad T^{12} = g^{11} T_1^2 = \underline{4} \quad T^{21} = g^{22} T_2^1 = \underline{1}$

$T^{22} = g^{22} T_2^2 = \underline{0}$

№1

$\omega = x^2 dx + xy^3 dy$

$d\omega = y^3 dx \wedge dy$

№3.

$\omega = \lambda x dy + y dx$

$d\omega = \lambda dx \wedge dy - dx \wedge dy = 0 \implies \lambda = 1$

№3.

$\omega = \dfrac{x dy - y dx}{x^2 + y^2} \qquad x = 2\cos\varphi \qquad \varphi^{-1}(\omega) = \dfrac{4\cos\varphi\sin\varphi + 4\cos\varphi\sin\varphi\, d\varphi}{4}$

$y = 2\sin\varphi$

$= 2\cos\varphi\sin\varphi\, d\varphi$

№6.

$\omega = x^2 \underset{0}{\underset{\parallel}{dx}} \wedge dy \qquad (x,y)\mapsto(0,x)$

№8.

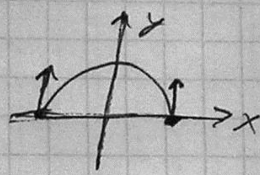
$X=(1,0) \qquad Y=(x^2+y,\ x-y)$

$[X,Y]^1 = X^1\cdot\frac{\partial Y^1}{\partial x} + 0 - 0 = 1\cdot 2x = \underline{2x}$

$[X,Y]^2 = X^1\cdot\frac{\partial Y^2}{\partial x} = \underline{1}$

№11.

$v=(0,1) \qquad x^2+y^2=1$



$\begin{cases} x=\cos\varphi \\ y=\sin\varphi \end{cases} \ \varphi\in[0,\pi]$

$\frac{dT^i}{dt} + 0 = 0 \qquad \rho T=(0,1)$

№13.

$f = yx^2+y^2 \qquad \nabla_i f = \frac{\partial f}{\partial x^i}$ — не интересно

№14.

$(x^2,y) \qquad T^1=x^2$

$T^2=y$

$(\nabla T)^1_1 = 2x$

$(\ )^1_2=0 \quad (\ )^2_1=0$

$(\nabla T)^2_2=1$

$g_{ij}=\begin{pmatrix}2&0\\0&3\end{pmatrix} \Rightarrow \Gamma^i_{jk}\equiv 0$

№13.

$(x-y^2,3y) \quad (\nabla T)^1_1=1 \quad (\nabla T)^2_1=0$

$(\nabla T)^1_2=-2y \quad (\nabla T)^2_2=3$