№ 1.

$T^i_j = \begin{cases} 0, & i \neq j \\ 2, & i = j \end{cases}$ $\qquad T^i_j = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix} = C \qquad C^{-1} = \frac{1}{-2}\begin{pmatrix} 3 & -1 \\ -5 & 1 \end{pmatrix}^T$

$\left(\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ 0 & -2 & -1 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 1 & -1 & 2 \\ 0 & -2 & -1 & 1 \end{array}\right) \to$

$T^{i'}_{j'} = \begin{pmatrix} -\frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix} =$ $\qquad \begin{pmatrix} -\frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = C^{-1}$

$= \begin{pmatrix} -3 & 5 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $\quad \forall T \in \mathbb{T}^1_1,\ T$ – операт.

№ 2.

$T^1_2 = 3 \quad T^1_1 = T^2_1 = T^2_2 = 0 \qquad T^2_1 \qquad C = \begin{pmatrix} 1 & 1 \\ 5 & 4 \end{pmatrix}$

$T^{2}_{1} = a^2_i d^j_1 T^i_j = a^2_1 d^2_1 3 = 2\,5 \cdot 3 = 75 \qquad C^{-1} = d = (-1)\begin{pmatrix} 4 & -5 \\ -1 & 1 \end{pmatrix}^T = \begin{pmatrix} -4 & 1 \\ 5 & -1 \end{pmatrix}$

$T^{i'}_{j'} = \begin{pmatrix} -4 & 1 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -12 \\ 0 & 15 \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 5 & -1 \end{pmatrix} =$

$= \begin{pmatrix} * & * \\ 15 \cdot 5 & * \end{pmatrix}$

$\boxed{75}\,!$

№ 3.

$\tilde{T}^{22}_{11} \qquad T = e^1 \otimes e^1 \otimes e_2 \otimes e_2 + 2 e^2 \otimes e^2 \otimes e_1 \otimes e_1 \qquad T^{22}_{11} = 1 \quad T^{11}_{22} = 2 \qquad a = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \quad d = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix}$

$\tilde{T}^{22}_{11} = a^2_i a^2_j d^k_1 d^l_1 T^{ij}_{kl} = a^2_2 a^2_2 d^1_1 d^1_1 T^{22}_{11} +$

$+ a^2_1 a^2_1 d^2_1 d^2_1 T^{11}_{22} = 1 \cdot 1 \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot 1 + 0 = \frac{1}{9}$

№ 4.

$T_{123} = 2 \quad T_{213} = T_{132} = T_{321} = -2 \qquad a = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

$T_{312} = T_{231} \quad T_{112} = \ldots = 0$

$d = (-1)\begin{pmatrix} 1 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^T = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$

$\tilde{T}_{123} = d^i_1 d^j_2 d^k_3 T_{ijk} = d^1_1 d^3_2 d^1_3 \underset{0}{\underbrace{T_{131}}} + d^2_1 d^3_2 d^1_3 T_{231} =$

$= 1 \cdot (-1) \cdot 1 \cdot 2 = -2$

$T = Alt(u\otimes v)$ № 5.

$u = 2e_1 + e_2$, $v = 2e_1$

$u\otimes v = 4e_1\otimes e_1 + 2e_2\otimes e_1$ $(u\otimes v)^{11} = 4$ $(u\otimes v)^{21} = 2$ ост $=0$

$T = \frac{1}{2}\sum_\sigma (-1)^\sigma \sigma(u\otimes v) = \frac{1}{2}\left(1\cdot(4e_1\otimes e_1 + 2e_2\otimes e_1) - 1\cdot(4e_1\otimes e_1 + 0)\right) = \underline{e_2\otimes e_1}$

№ 6.

$T = Sym(\xi\otimes\eta)$ $\xi = -e^1$, $\eta = e^2 - e^1$

$\xi\otimes\eta = -e^1\otimes e^2 + e^1\otimes e^1$

$T = \frac{1}{2}\sum_\sigma \sigma(\xi\otimes\eta) = \frac{1}{2}\left((-e^1\otimes e^2 + e^1\otimes e^1) + (e^1\otimes e^1)\right) = e^1\otimes e^1$

№ 7.

$\xi = e^1 + e^2$, $\eta = e^1 - e^2$

$(\xi\otimes\eta)(5e_1, 4e_2) = (e^1\otimes e^1 - e^1\otimes e^2 + e^2\otimes e^1 - e^2\otimes e^2)(5e_1, 4e_2) =$

$= \underline{-20}$

№ 8.

T: $2e_1\otimes e^2 - 3e_2\otimes e^2 + e_3\otimes e^2 - e_3\otimes e^3$ $C_1^1(T) = \underline{-4}$

№ 9.

$T = e_3\otimes e_2\otimes e_1 + e_1\otimes e_1\otimes e_2$ $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

$\sigma T = \cancel{e_3\otimes e_1\otimes e_2 + e_2\otimes e_2\otimes e_1}$

$\| \; e_2\otimes e_3\otimes e_1 + e_1\otimes e_1\otimes e_2$

№ 10. $T_{12} = 2$ $T_{21} = 3$

$T = 2e^1\otimes e^2 + 3e^2\otimes e^1$ $g^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$T^i_j = g^{ik}T_{kj}$ $T^1_2 = g^{11}T_{12} = 1\cdot 2 = 2$

$T^2_1 = g^{22}T_{21} = 2\cdot 3 = 6$

№ 11.

$\underbrace{dx + 2dy}$, $\underbrace{x^2dy\wedge dz + dz\wedge dx}$

$u\wedge v = \frac{(p+q)!}{p!\,q!}Alt(u\otimes v)$

$u\otimes v = dx\otimes(x^2dy\wedge dz$

№12.

$$\omega = x^2dx + xy^3dy$$

$$d\omega = y^3dx\wedge dy$$

№13.

$$\omega = 3\lambda x\,dy\wedge dz + y\,dx\wedge dz$$

$$d\omega = 3\lambda\,dx\wedge dy\wedge dz - 1\cdot dx\wedge dy\wedge dz = 0 \qquad 3\lambda = 1 \qquad \lambda = \frac{1}{3}$$

№17.

$$X = (y, 0) \qquad Y = (x^2+y,\ x-y)$$

$$[X,Y]^i = X^j\frac{\partial Y^i}{\partial x^j} - Y^j\frac{\partial X^i}{\partial x^j}$$

$$[X,Y]^1 = X^1\frac{\partial Y^1}{\partial x} - \underbrace{Y^1\frac{\partial X^1}{\partial x}}_{0} - Y^2\frac{\partial X^1}{\partial y} = y\cdot 2x - (x-y)\cdot 1 = 2xy+(y-x)$$

$$[X,Y]^2 = X^1\frac{\partial Y^2}{\partial x} - 0 = y\cdot 1 = y$$

№18.

$$X = (0,0,1) \qquad Y = (x-3y,\ -y^3,\ z)$$

$$[X,Y]^1 = X^3\frac{\partial Y^1}{\partial z} = 0 \qquad [X,Y]^2 = 0 \qquad [X,Y]^3 = 1$$

№19.

$$f = yx^2 + y^2 \qquad \nabla_x f = 2xy$$

$$\nabla_y f = x^2 + 2y$$

№20.

$$v = (x^2, y) \qquad dS^2 = 2dx^2 + 3dy^2 \qquad \nabla_x v^1 = 2x \qquad \nabla_y v^1 = 0$$

$$\nabla_x v^2 = 0 \qquad \nabla_y v^2 = 1$$

$$\Gamma^{i'}_{j'k'} = \frac{\partial x^{i'}}{\partial x^{n}} \cdot \frac{\partial^2 x^{n}}{\partial x^{j'} \partial x^{k'}}$$

№ 22.2

$$\begin{cases} u = y \\ v = xy \end{cases} \qquad \begin{cases} x = \frac{v}{u} \\ y = u \end{cases} \qquad x''_{u} = \left(-\frac{v}{u^2}\right)' = 2\frac{v}{u^3}$$

$$\Gamma^{2}_{11} = \frac{\partial v}{\partial x} \cdot \frac{\partial^2 x}{\partial u \partial u} + \frac{\partial v}{\partial y} \frac{\partial^2 y}{\partial u \partial u} = y \cdot 2\frac{v}{u^3} + x \cdot 0 = 2\frac{v}{u^2}$$

№ 21.1

$$\Gamma^{1}_{22} = \frac{1}{2} \cdot g^{11}\left(\frac{\overset{0}{\partial g_{21}}}{\partial x^1} + \frac{\overset{0}{\partial g_{12}}}{\partial x^1} - \frac{\partial g_{22}}{\partial x}\right) = \qquad G = \begin{pmatrix} x^2 & 0 \\ 0 & x^2 \end{pmatrix}$$

$$= \frac{1}{2} \cdot \frac{1}{x^2}(-2x) = -\frac{1}{x}$$

№ 23.

$\Gamma^{1}_{11} = y \quad \Gamma^{2}_{11} = \Gamma^{2}_{22} = 1 \quad$ ост-е $= 0 \quad \underline{y = 3} \quad \gamma \begin{cases} x = t \\ y = 3 \end{cases}$

$$\frac{dT^i}{dt} + \Gamma^{i}_{\beta\alpha} T^{\beta} \dot{x}^{\alpha} = 0$$

$$\begin{cases} \frac{dT^1}{dt} + \Gamma^{1}_{11} \cdot T^1 \cdot \overset{1}{\dot{x}} = 0 \\ \frac{dT^2}{dt} + \Gamma^{2}_{11} T^1 \dot{x} + \Gamma^{2}_{22} T^2 \overset{0}{\dot{y}} = 0 \end{cases}$$

$$\frac{dT^1}{dt} + y \cdot T^1 = 0$$

$$\frac{dT^1}{T^1} = -y\,dt \qquad \underline{T^1 = C_1 e^{-yt}} \qquad \frac{dT^2}{dt} + T^1 = 0$$

$$dT^2 = -C_1 e^{-yt} dt$$

$$\underline{T^2 = C_1 \frac{1}{y} e^{-yt} + C_2}$$

№ 24.

$$\Gamma^{2}_{11} = 2 \quad \Gamma^{1}_{12} = \Gamma^{1}_{21} = y \qquad \ddot{u}^i + \Gamma^{i}_{\alpha\beta} \dot{u}^{\alpha} \dot{u}^{\beta} = 0$$

$$\begin{cases} \ddot{x} + \Gamma^{1}_{12} \cdot \dot{x}\dot{y} + \Gamma^{1}_{21} \dot{y}\dot{x} = 0 \\ \ddot{y} + \Gamma^{2}_{11} \dot{x}\dot{x} = 0 \end{cases} \qquad \begin{cases} \ddot{x} + 2\dot{x}\dot{y} = 0 \\ \ddot{y} + 2\dot{x}^2 = 0 \end{cases} \qquad \dot{x} = p \quad \dot{y} = q$$

$$\begin{cases} \dot{p} + 2pq = 0 \\ \dot{q} + 2p^2 = 0 \end{cases}$$

$$q = -\frac{\dot{p}}{2p}$$

$$\dot{q} =$$